Lecture 21 Summary

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1 Forces on Vortices

After finding the force exerted by one vortex on another it is possible to calculate the force exerted on a single vortex by a transport current $\overrightarrow{J}$. This situation arises, for example, in a high field magnet when the field created by the current enters the superconducting wire. Imagine an infinite line of parallel vortices and look at the net current produced by all the vortices at a point some perpendicular distance away. All of those current contributions will super-impose to create a current flowing parallel to the line. Placing a test vortex there will create a net force which is given by,
$\overrightarrow{f} = \overrightarrow{J} \times \Phi_0 \hat{z}$.
Again the vortex will move perpendicular to the applied current in a Lorentz force-like manner. As the vortex moves there is a time-rate-of-change of the magnetic field. By Faraday's law this $\partial\overrightarrow{B}/\partial t$ gives rise to an electric field as $\nabla \times \overrightarrow{E} = -\partial\overrightarrow{B}/\partial t$. If the flux is out of the page and the current is flowing from left to right, the force exerted on the vortex is downward. As the vortex moves $-\partial\overrightarrow{B}/\partial t$ points out of the page at the instantaneous location of the vortex and $-\partial\overrightarrow{B}/\partial t$ points in to the page just below there in the direction of motion. These contributions both give an electric field pointing to the right, in the direction of the current. This will produce Ohmic losses in the normal core of the vortex, resulting in dissipation as the vortex moves. The superconductor now has a non-zero dc resistance. Finding ways to prevent the vortices from moving under such circumstances is essential to restore the zero-resistance properties of superconductors in a magnetic field. This is the subject of "vortex pinning".

Now imagine a collection of vortices being acted upon by a uniform current $\overrightarrow{J}$. As the vortices move the electric field induced is given by $\overrightarrow{E} = \overrightarrow{B} \times \overrightarrow{v}_v$, where $\overrightarrow{v}_v$ is the vortex velocity. This electric field is parallel to $\overrightarrow{J}$. Write the magnetic flux density as $|\overrightarrow{B}| = n_v\Phi_0$, where n_v is the number of vortices per unit area. Energy will be dissipated at a rate of $W = \overrightarrow{J} \cdot \overrightarrow{E}$.

Where does the dissipated energy go? The Bardeen-Stephen model says that it is dissipated by inducing currents in the normal core of the vortex. The power dissipated by a single vortex is $\left(\pi a^2 L_z\right) \rho_n J^2$, where a is the radius of the vortex core, L_z is the length of the vortex in the superconductor, ρ_n is the resistivity of the electron fluid in the vortex core, often taken to be the normal state resistivity of the metal. The power dissipated in the entire sample is $\left(\pi a^2 L_z\right) \rho_n J^2 n_v A$, where A is the area of the sample. Finally, the power dissipated per unit volume of the sample is $\pi a^2 \rho_n J^2 n_v$. Equating this to $W = JE$ yields an expression for the flux-flow resistivity: $E = \rho_{ff} J$, with $\rho_{ff} = \pi a^2 \rho_n \frac{B}{\Phi_0}$. Recall that $\mu_0 H_{c2} = \frac{\Phi_0}{2\pi \xi_{GL}^2}$, so taking $a \approx \xi_{GL}$ gives $\rho_{ff} \approx \rho_n \frac{B}{B_{c2}}$. Hence the flux flow resistivity is the normal state resistivity times the fractional coverage of the sample with vortex cores. This prediction for the magneto-resistance of a superconductor is in generally good agreement with data for low-temperature superconductors, as shown on the class web site.

The flux flow resistivity can be associated with a vortex viscous force. Solving for the vortex velocity in terms of the flux flow resistivity above yields $v_v = \rho_n J / B_{c2}$. The fact that the vortex velocity scales with the applied current implies an equilibrium between a driving force and a dissipative force. We call this latter force the viscous drag force on the vortex, $\overrightarrow{f}_{drag} = -\eta \overrightarrow{v}_v$ with $\eta = \frac{J\Phi_0}{v_v}$, or $\eta = \frac{\Phi_0}{\rho_n} B_{c2}$.
The equation of motion for a single vortex acted upon by a current is then $\overrightarrow{J} \times \Phi_0 \widehat{z} - \eta \overrightarrow{v}_v + \overrightarrow{F}_{pin} = 0$, where $\overrightarrow{F}_{pin}$ is the pinning force on the vortex.

2 Vortex Pinning

Any region in space where the magnitude of the superconducting order parameter is reduced is a potential pinning site. If there is a region with $|\psi| = 0$ of length D_z, then the free energy gain of locating the vortex core there is $\Delta F_0 = \frac{\mu_0 H_c^2}{2} \pi \xi_{GL}^2 D_z$. Removing the vortex core from this location requires a finite energy. As long as the vortex is stationary in the presence of a current there will be no energy dissipation. This means that the superconductor has a new kind of critical current, dictated by the strength of the vortex pinning.
Pinning can be thought of as creating a Hooke's law restoring force for the vortex as $\overrightarrow{F}_{pin} = -k(\overrightarrow{x} - \overrightarrow{x}_0)$. The fine art of vortex pinning is to make the superconductor in to "Swiss cheese" by suppressing the order parameter in limited cylindrical regions to pin the cores, but leaving the remainder un-touched so that it can support the screening currents. In this way one can have a zero resistance state in high magnetic field in the presence of a transport current, but only up to a point in either temperature, current or magnetic field.
Low-Tc superconductors can display "collective pinning" if they have a sufficiently rigid vortex lattice structure. In this case, pinning a single vortex effectively pins the entire lattice.
One can also have the phenomenon of Thermally-Assisted Flux Flow (TAFF)

in which vortices move between different pinning sites in a disordered pinning landscape due to thermally-activated motion.

3 The Josephson Effect

Josephson tunneling of Cooper pairs takes place between two superconductors separated by a "weak link" in which the order parameter is suppressed. Many varieties of weak links exist, but it is easiest to do the calculation for the case of an insulating barrier between the two superconductors (SIS tunneling).
The superconductors have macroscopic quantum wavefunctions given by $\Psi_1 = \sqrt{n_1^*}e^{i\theta_1}$ and $\Psi_2 = \sqrt{n_2^*}e^{i\theta_2}$, and the barrier between them has thickness $2a$.
Start with the time-independent Schrodinger equation for Ψ and the equation for the the current:
$\Lambda \overrightarrow{J}_s(\overrightarrow{r}, t) = \frac{\hbar}{e^*}\overrightarrow{\nabla}\theta - \overrightarrow{A}(\overrightarrow{r}, t)$ with $\Psi(\overrightarrow{r}, t) = \sqrt{n^*}e^{i\theta(\overrightarrow{r}, t)}$.
Make two assumptions:
1) The junction area is small so that the current density is uniform across the junction. In other words the area is small compared to λ_{eff}^2. This makes the problem one-dimensional.
2) Take the magnetic $\overrightarrow{A} = 0$ and electric $\phi = 0$ fields to be zero. These will be added back in later.

One now has a standard quantum barrier tunneling problem with solution
$J_s = -\frac{e^*\hbar}{m^*\zeta}\frac{\sqrt{n_1^* n_2^*}}{2\sinh(2a/\zeta)}\sin(\theta_1 - \theta_2)$,
where $\zeta^2 = \frac{\hbar^2}{2m^*(V_0 - E)}$, and $V_0 - E$ is the barrier height. The prefactor is the critical current density of the junction,
$J_c = \frac{e^*\hbar}{m^*\zeta}\frac{\sqrt{n_1^* n_2^*}}{2\sinh(2a/\zeta)}$ and depends on the geometry of the junction as well as the superconductors involved. For thick insulators this reduces to,
$J_c = \frac{e^*\hbar}{m^*\zeta}\frac{\sqrt{n_1^* n_2^*}}{2}e^{-2a/\zeta}$.
The exponential dependence of critical current on barrier thickness and height makes it extremely difficult to make large numbers of Josephson junctions with identical properties, a necessary requirement for applications such as large-scale computing.
The critical current will have the same temperature dependence as the superfluid: $J_c(T) \propto n_s(T)$ as $T \to T_c$.
This result also suggests that the Josephson current-phase relationship is $\sin(\theta_1 - \theta_2)$, which is found to be correct in many low-Tc Josephson junctions (JJs), but deviations from this simple sinusoidal dependence are seen in disordered d-wave JJs, as illustrated on the class web site.

Bardeen and Josephson had a fundamental disagreement about Cooper pair tunneling through an insulating barrier. Bardeen (using the BCS k-space picture) believed that since $V_{k,k'} = 0$ in the insulator, there could be no support for Cooper pairs and therefore such tunneling was incoherent. If the tunneling probability for a single particle is t (with $t << 1$) then the tunneling probability for a Cooper pair is t^2, and therefore will be swamped by quasiparticle tunneling. Josephson was following the work on generalization of BCS to real space, where it was predicted that the pair potential $\Delta(r)$ was non-zero in the insulator. Therefore he wrote down a tunneling Hamiltonian in which pair tunneling swamped the quasiparticle tunneling. Josephson turned out to be correct, and he won the Nobel prize in physics the year after BCS did for their theory of superconductivity.